Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2015

Name:________________________________

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Exam II Solutions

March 5, 2015

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

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Place an × through your answer to each problem.

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Multiple Choice

1. (5 pts.) An experiment consists of rolling two 6-sided dice and recording the pair of numbers rolled. Consider the following events

$$\begin{aligned} E &= \text{event at least one of the numbers is a 6} \\ F &= \text{event both numbers are odd} \\ G &= \text{event at least one of the numbers is a 3} \\ H &= \text{event the numbers add to 8} \end{aligned}$$

Which of the following is a mutually exclusive pair of events?

(a) E and F (b) E and G (c) G and H

(d) F and G (e) E and H

Solution:
Consider each of the answer choices

- E and F. We cannot have both numbers be odd and have one of the numbers be a 6, so $E \cap F = \emptyset$ and E and F are mutually exclusive.
- E and G. The outcome $(6, 3)$ is in $E \cap G$, so the events are not mutually exclusive.
- G and H. The outcome $(5, 3)$ is in $G \cap H$, so the events are not mutually exclusive.
- F and G. The outcome $(3, 1)$ is in $F \cap G$, so the events are not mutually exclusive.
- E and H. The outcome $(6, 2)$ is in $E \cap H$, so the events are not mutually exclusive.

2. (5 pts.) An experiment has sample space $S = \{a, b, c, d, e\}$. The following table gives the probabilities for these outcomes, except $P(c)$ is unknown.

Outcome	**Probability**
a	0.20
b	0.10
c	x
d	0.25
e	0.30

If we define the event $E = \{a, b, c\}$, what is $P(E)$?

(a) 0.70 (b) 0.15 (c) 0.55

(d) 0.45 (e) 0.85

Solution: Here are two possible solutions:

(1) Remember that the probabilities of all of the outcomes in the sample space must add to 1. Therefore, $0.20 + 0.10 + x + 0.25 + 0.30 = 1$, so $P(c) = x = 1 - 0.20 - 0.10 - 0.25 - 0.30 = 0.15$. Thus,

$$P(E) = P(a) + P(b) + P(c) = 0.20 + 0.10 + 0.15 = 0.45.$$

(2) For a second solution, notice that $E' = \{d, e\}$. Therefore, we have

$$P(E) = 1 - P(E') = 1 - (P(d) + P(e)) = 1 - (0.25 + 0.30) = 1 - 0.55 = 0.45.$$

3. (5 pts.) A coin is flipped 12 times and the sequence of heads and tails is recorded. If $E =$ the event that exactly 6 tails are obtained, what is $P(E)$?

(a) $C(12, 6) \cdot 2^{12}$ (b) $\frac{1}{2}$ (c) $\frac{6}{2^{12}}$

(d) $C(12, 6)$ (e) $\frac{C(12, 6)}{2^{12}}$

Solution:

Our sample space is all sequences of 12 H's and T's, for example $HTHHTHTTHTHT$ is an element. Notice that our sample space contains equally likely outcomes, the likelihood of one outcome is the same as the likelihood of any other outcome. Therefore, we have

$$P(E) = \frac{n(E)}{n(S)} = \frac{C(12, 6)}{2^{12}}$$

4. (5 pts.) Of a group of 100 students, 60 like downhill skiing, 40 like cross country skiing, and 80 like at least one of the two. A student is chosen at random. If it is known that the student chosen likes cross country skiiing, what is the probability he or she likes downhill skiing?

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{2}$

(d) $\frac{1}{5}$ (e) $\frac{1}{3}$

Solution:

Let D be the event the student likes downhill skiing, and let C be the event the student likes cross country skiing. Then the probability the student likes downhill skiing given that he or she likes cross country skiing is

$$P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{P(D) + P(C) - P(D \cup C)}{P(C)} = \frac{\frac{60}{100} + \frac{40}{100} - \frac{80}{100}}{\frac{40}{100}} = \frac{\frac{20}{100}}{\frac{40}{100}} = \frac{20}{40} = \frac{1}{2}.$$

5. (5 pts.) Three cards are drawn from a standard deck (without replacement). What is the probability that the first card is red and the second and third cards are black? (All answer choices are rounded to three decimal places.)

(a) 0.382 (b) 0.127 (c) 0.125

(d) 0.120 (e) 1.510

Solution:

Let $R1$ be the event the first card is red, let $B2$ be the event the second card is black, and let $B3$ be the event the third card is black. Then the probability the first card is red and the second and third cards are black is

$$P(R1 \cap B2 \cap B3) = P(R1 \cap B2)P(B3|R1 \cap B2) = P(R1)P(B2|R1)P(B3|R1 \cap B2) = \frac{26}{52} \cdot \frac{26}{51} \cdot \frac{25}{50} = 0.127.$$

You can get this by using the formula for the probability of an intersection or by drawing a tree diagram.

6. (5 pts.) A club has 10 members, of which 5 are Math majors and 4 are English majors. A student is chosen at random and asked to state her major(s). Let M be the event that the student is a Math major and E be the event that the student is an English major. How many students must be majoring in *both* Math and English in order for M and E to be independent events? (Hint: What is $P(M)$? What is $P(E)$?)

(a) 0 (b) 2 (c) 1 (d) 3 (e) 4

Solution:

$P(M) = \frac{5}{10} = \frac{1}{2}$. $P(E) = \frac{4}{10} = \frac{2}{5}$. In order for M and E to be independent, we must have

$$P(M \cap E) = P(M) \cdot P(E) = \frac{1}{2} \cdot \frac{2}{5} = \frac{2}{10}.$$

Thus two students must be majoring in both math and English.

7. (5 pts.) Bob and Mary are having a basketball free-throw shooting contest. Bob gets a basket on 20% of his attempts, and Mary gets a basket on 30% of her attempts. (They're not very good free-throw shooters!) If they each make two attempts (for a total of four attempts between them) what is the probability that all four attempts are **misses**? Assume that the attempts are independent of each other. Round to the nearest 0.1 percent.

(a) 31.4% (b) 68.6% (c) 0.4% (d) 56% (e) 6%

Solution:
Let B be the even that Bob makes his free throw, and let M be the event that Mary makes hers. So $P(B) = 0.2$ and $P(M) = 0.3$. Then B' is the event that Bob misses his free throw, and M' is the event that Mary misses hers. Thus the probability that all four free throws are missed, to the nearest 0.1%, is

$$(0.8)(0.8)(0.7)(0.7) = 31.4\%.$$

8. (5 pts.) A store sells three brands of batteries: Ajax, Batterymundo and Cheap-O. Not being of very high quality, $\frac{1}{8}$ of the Ajax batteries are defective, $\frac{1}{4}$ of the Batterymundo batteries are defective, and $\frac{3}{8}$ of the Cheap-O batteries are defective. The store sells equal numbers of all three batteries. Bob chooses a battery at random from the store. If it turns out to be defective, what is the probability that it was a Cheap-O battery? (Answers below are given to three decimal places.)

(a) $\dfrac{1}{8} = 0.125$ (b) $\dfrac{3}{8} = 0.375$ (c) $\dfrac{1}{3} = 0.333$

(d) $\dfrac{1}{4} = 0.250$ (e) $\dfrac{1}{2} = 0.500$

Solution:
Let A, B and C be the events that Ajax, Batterymundo and Cheap-O (respectively) are chosen. Let D be the event that the chosen battery is defective, so D' is the event that it's not. We want $P(C|D)$.

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{P(D|C) \cdot P(C)}{P(D)} = \frac{\frac{3}{8} \cdot \frac{1}{3}}{(\frac{1}{8} \cdot \frac{1}{3}) + (\frac{1}{4} \cdot \frac{1}{3}) + (\frac{3}{8} \cdot \frac{1}{3})} = \frac{(\frac{1}{8})}{(\frac{6}{24})} = \frac{1}{2}.$$

9. (5 pts.) The following table shows the distribution of grades on a recent English exam, as percentages of the class of 40 students.

Grade	**Percentage**
0 – 59	10%
60 – 69	20%
70 – 79	30%
80 – 89	15%
90 – 100	25%

How many students scored between 70 and 89 (inclusive) on the exam? (Note that we are asking for the number of students, not the percentage.)

(a) 9 (b) 45 (c) 20 (d) 18

(e) Can't be determined from the table

Solution:
The percentage is $30 + 15 = 45\%$. There are 40 students, so the number is 45% of 40, or 18.

10. (5 pts.) The 10 students in an Italian Literature class got the following scores on a recent exam (listed in increasing order):

$$84,\ 85,\ 85,\ 85,\ 89,\ 90,\ 93,\ 96,\ 96,\ 99.$$

What was the median score on the exam?

(a) 89 (b) 89.5 (c) 90 (d) 85 (e) 90.2

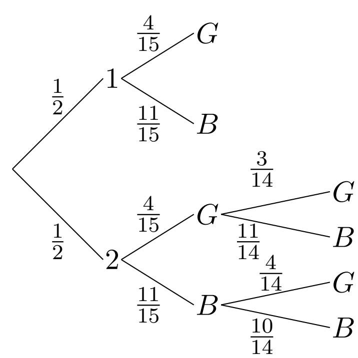
Solutions:
Halfway between 89 and 90, i.e. 89.5.

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) Andrew plays the following game. First he flips a coin with a 1 painted on one side and a 2 painted on the other. The number on the coin determines how many marbles he gets to draw (without replacement) from an urn containing 11 blue marbles and 4 green marbles (he draws the marbles in succession, not at the same time). Once he's drawn the allowed number of marbles, he wins if at least one of the marbles drawn is green.

(a) Draw a tree diagram to represent the game. All branches of the diagram should be labeled with probabilities.
Solution:



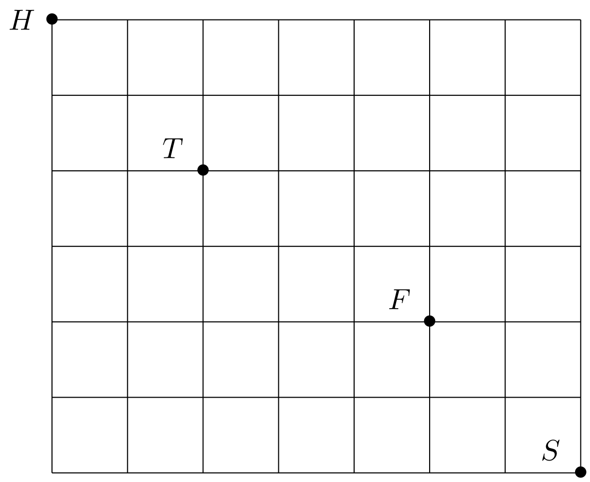
(b) What is the probability of winning? (You do not need to simplify your answer.)
Solution:
We're told that Andrew wins if at least one of the marbles drawn is green. Of the 6 branches in the tree diagram, this happens in branches 1, 3, 4, and 6 (counting from the top). Thus, we add the probabilities of those branches.

$$P(\text{winning}) = \left(\frac{1}{2}\right)\left(\frac{4}{15}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{15}\right)\left(\frac{3}{14}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{15}\right)\left(\frac{11}{14}\right) + \left(\frac{1}{2}\right)\left(\frac{11}{15}\right)\left(\frac{4}{14}\right)$$

(c) What is the probability that Andrew wins given that he gets a 2 on the coin flip? (You do not need to simplify your answer.)
Solution:
Let W be the event that Andrew wins, and let T be the event that he gets a 2 on the coin flip, then using the tree diagram and the formula for conditional probability gives

$$P(W|T) = \frac{P(W \cap T)}{P(T)} = \frac{\left(\frac{1}{2}\right)\left(\frac{4}{15}\right)\left(\frac{3}{14}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{15}\right)\left(\frac{11}{14}\right) + \left(\frac{1}{2}\right)\left(\frac{11}{15}\right)\left(\frac{4}{14}\right)}{\frac{1}{2}}$$

12. (10 pts.) For this problem you do not need to simplify your answers; you may leave permutations, combinations, and factorials (i.e. $P(n,k)$, $C(n,k)$, $\binom{n}{k}$, or $n!$) or products, sums, or quotients of numbers, permutations, combinations, or factorials in your answers if you so choose. The following is a map of Carl's neighborhood.



Carl wants to walk from his house at H to the store at S without backtracking (he travels only south and east). He chooses a route at random.

(a) What is the probability Carl passes his friend Fred's house at F?
Solution:
The sample space is the set of routes from H to S, and since Carl chooses a route at random, the outcomes are equally likely. Therefore, if F is the event Carl passes Fred's house,

$$P(F) = \frac{n(F)}{n(S)} = \frac{C(9,4)C(4,2)}{C(13,6)}$$

(b) If Carl's friend Taylor lives at T, what is the probability Carl passes both Fred's house and Taylor's house?
Solution:

$$P(\text{Carl passes both houses}) = \frac{\text{number of routes that pass both houses}}{\text{total number of routes}} = \frac{C(4,2)C(5,2)C(4,2)}{C(13,6)}$$

(c) What is the probability Carl passes at most one of his friends' houses?
Solution:

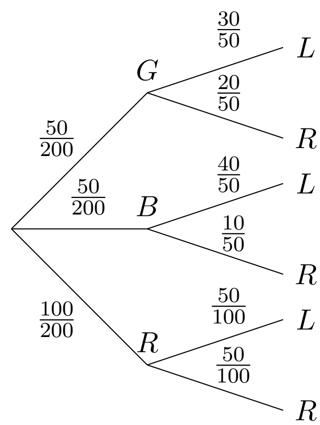
$$P(\text{Carl passes at most one house}) = 1 - P(\text{Carl passes both houses}) = 1 - \frac{C(4,2)C(5,2)C(4,2)}{C(13,6)}$$

13. (10 pts.) A college bookstore sells three kinds of T-shirts: green, blue and red. For each kind, they sell small and large T-shirts. On a given day, they have 200 T-shirts in stock, all mixed together in a large bin and divided as follows:

Color	**Number Small**	**Number Large**
Green	20	30
Blue	10	40
Red	50	50

(a) Draw a tree diagram to represent this situation with the branches labelled as probabilities (e.g. the 50 greens would correspond to the probability $P(G) = \frac{50}{200} = 25\%$ and the 30 large greens would correspond to the conditional probability $P(L|G) = \frac{30}{50} = 60\%$).

Solution:



(b) During the night a thief breaks in and randomly steals one T-shirt from the bin. What is the probability that he stole a red T-shirt? Explain your answer.

Solution:
There are 100 red T-shirts and 200 T-shirts all together, so the probability is $\frac{1}{2}$.

(c) Before he gets into the light he tries it on and realizes that it is a large T-shirt. With this additional information, what is the probability that he stole a red T-shirt? Explain your answer. You do not need to simplify your answer.

Solution:

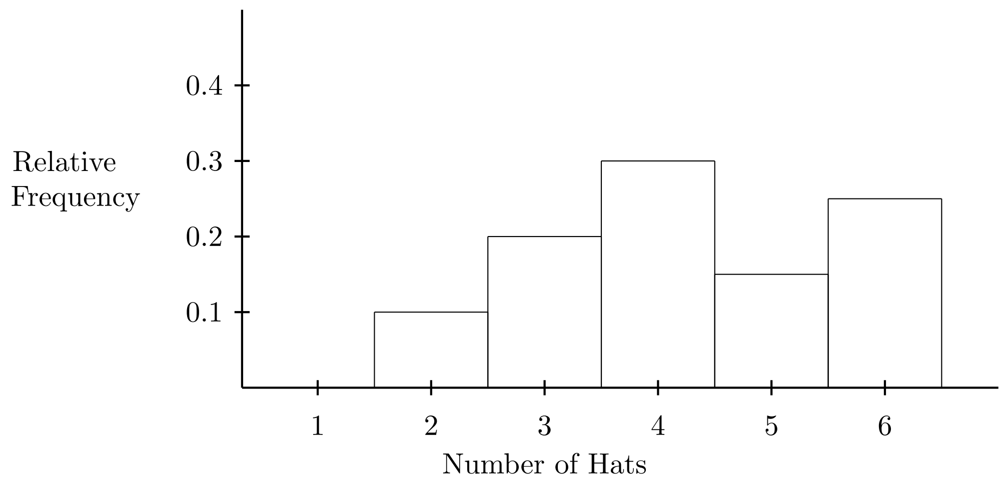
$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{(\frac{1}{4} \cdot \frac{3}{5}) + (\frac{1}{4} \cdot \frac{4}{5}) + (\frac{1}{2} \cdot \frac{1}{2})} = \frac{5}{12}.$$

14. (10 pts.) In a certain club the members are each asked how many hats they own, and the responses are as follows (given as relative frequencies):

Number	**Relative Frequency**
2	$0.10 = 10\%$
3	$0.20 = 20\%$
4	$0.30 = 30\%$
5	$0.15 = 15\%$
6	$0.25 = 25\%$

(a) Draw a histogram to represent this information.

Solution:



(b) What is the **mean** number of hats owned? Show your work!

Solution:

$$(2)(.1) + (3)(.2) + (4)(.3) + (5)(.15) + (6)(.25) = 4.25.$$

(c) What is the **median** number of hats owned? Be sure to explain your answer.

Solution:

The middle mark comes at 50%, which is in the range where the people own 4 hats (which goes from 30% to 60%).

15. (10 pts.) Let A, B and C be events and assume the following probabilities:

$$P(A) = 0.2, \quad P(B) = 0.3, \quad P(C) = 0.5, \quad P(A \cap B) = 0.06, \quad P(A \cap C) = 0, \quad P(B \cap C) = 0.5.$$

(a) Which two of the three events are independent? Explain.

Solution:
There was actually a logical mistake in this problem, although it did not affect the computation of the solution. It's impossible for $P(B \cap C)$ to be 0.5 when $P(B)$ is only 0.3. Nevertheless, we have $P(A) \cdot P(B) = P(A \cap B)$, so A and B are independent.

(b) Which two of the events are mutually exclusive? Explain.

Solution:
A and C are mutually exclusive since their intersection is empty.

(c) What is the conditional probability $P(B|C)$?

Solution:

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0.5}{0.5} = 1.00.$$

This means that if C occurs, you are guaranteed that B will occur.

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